On the Robustness of Multidimensional Poverty Orderings in the EU

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Questions

The measurement of poverty is complex: many methodological and normative choices

Especially true with multidimensional measures

1. Under which conditions can we claim *robustly* that poverty in region A is higher than in region B?

2. When can we say that poverty in a given region has *unambigously* declined or increased?

Introduction

- In the multidimensional poverty measurement: *weights* affect the identification and the depth of poverty
- How should we weigh the dimensions? No consensus
- Standard approach: equal weights and then robustness checks for a grid of vectors, e.g. official measure in the EU
- Not a good idea: poverty comparisons are in general extremely sensitive to weights
- Importance of dominance conditions

Measurement Framework

- Alkire and Foster (2011): very influential paper
- Counting approach to multidimensional poverty: valid for cardinal and ordinal dimensions
- Assume N units and D indicators of wellbeing. If $x_{nd} < z_d$ then n is deprived in d
- For each dimension $\exists\ w_d \in (0,1)$ such that $\sum_{d=1}^{D} w_d = 1$
- The deprivation score for each individual is:

$$c_n \equiv \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$$

Measurement Framework

- Identification rule $\rho(W, k)$: a person is deemed poor: $c_n \geq k$, where $k \in [0, 1]$
- Individual poverty function $\begin{cases} p_n = \mathbb{I}(c_n \geq k) g(c_n) & \text{if } n \text{ is poor} \\ 0 & \text{otherwise} \end{cases}$
- where $g(c_n)$ satisifies $g(0) = 0$, $g' > 0$
- The following class of poverty indices

$$P(W, k) = \frac{1}{N} \sum_{n=1}^{N} p_n$$

Necessary and Sufficient Condition

Condition 1 $P^A < P^B$ for all P in $\mathbb{P}_1$ and any identification cut-off, k, if and only if
$H^A(k) \leq H^B(k) \;\; \forall k \in [0, v_2, ..., 1] \;\; \wedge \exists k | H^A(k) < H^B(k)$.

Condition 2 Consider the class of poverty measures $\mathbb{P}_1$. The following three statements are equivalent:

1. $P^A < P^B$ for all $P \in \mathbb{P}_1$ for any weighting vector, W, and poverty threshold, k.
2. For any vector of weights, W,
$H^A(k) \leq H^B(k) \;\; \forall k \in [0, v_2, ..., 1] \wedge \exists k | H^A(k) < H^B(k)$.
3. For all $\gamma_{W,k} \in \Gamma$, $\Pi(W, k)$ in A is no greater than in B, and at least once strictly lower.

- **People at risk of poverty or social exclusion:** one of the Europe 2020 Strategy headline indicators (to monitor progress towards the Europe 2020 strategy targets) adopted in 2010.
- Defined as the sum of persons who are:
  - at-risk-of-poverty and/or
  - severely materially deprived and/or
  - living in households with very low work intensity.
- In terms of A-F family of poverty measures, EU multidimensional poverty indicator has a form of headcount ratio $H(k, \mathbf{w})$ with $w_1 = w_2 = w_3 = \frac{1}{3}$ and $k = \frac{1}{3}$, i.e. the *union approach.*

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- **Data Sources:** EU-SILC (European Union Statistics on Income and Living Conditions)
  - Time period: 2004 - 2013
  - Spatial coverage: EU-28 plus: Iceland, Switzerland and Norway
  - Sample sizes (complete observations): almost 6.5mil in total, ranging from 8,545 (IS-2009) to 61,542 (IT-2004)
- Living in households with equivalised disposable income **below 60 %** of the national equivalised median income (after social transfers).
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- Living in a household that cannot afford to pay for **at least four** out of nine items:
  1. to face unexpected expenses
  2. one week annual holiday away from home
  3. to pay for arrears (mortgage or rent, utility bills or hire purchase installments)
  4. a meal with meat, chicken or fish every second day
  5. to keep home adequately warm,

  or could not afford (even if wanted to):

  6. a washing machine
  7. a colour TV
  8. a telephone
  9. a personal car

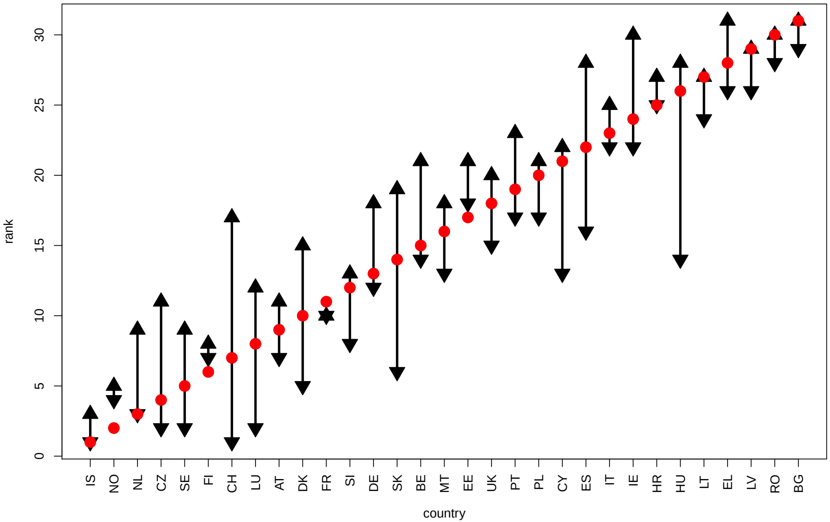
- Living in a household, where working-age adults (18-59) worked **less than 20 %** of their total work potential during the past year.
  - Based on the number of “months at work” and “workable months” of working age persons (18-64) in the household.

Do Weights Matter? #1

- In empirical literature sensitivity to weights checks are usually based on a very limited number of weighting schemes, e.g.:
  - Assigning a weight of e.g. 0.5 to one of the dimensions and 0.25 to each of the remaining two dimensions.
  - Assessment based e.g. on Spearman's ρ.
- Applying that approach to 2012 EU data we get the following results:
  - $(w_1 = 0.5, w_2 = 0.25, w_3 = 0.25) : \rho_s = \mathbf{0.879}$
  - $(w_1 = 0.25, w_2 = 0.5, w_3 = 0.25) : \rho_s = \mathbf{0.949}$
  - $(w_1 = 0.25, w_2 = 0.25, w_3 = 0.5) : \rho_s = \mathbf{0.939}$

BUT:

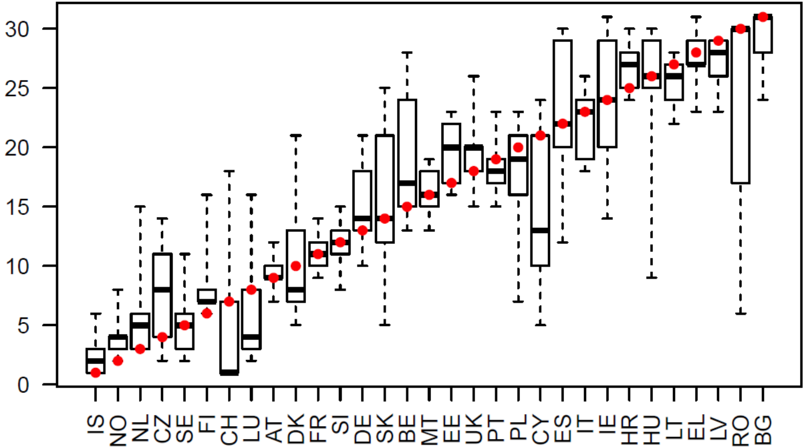
Ranks for the “common” weighting schemes



Do Weights Matter? #2

- Q: How do ranks change if we consider a wider range of combinations of k's and $\mathbf{w}$'s?
- Simple simulation:
  - $k = \left(\frac{1}{60}, \frac{2}{60}, \ldots, 1\right)_{N=60}$
  - Creation of weighting vectors $\mathbf{w}$ are based on *permutations with repetition* of elements of $\mathbf{w_0} = \left(\frac{1}{60}, \frac{2}{60}, \ldots, \frac{59}{60}\right)$ for which $\sum_{i=1}^{3} w_i = 1$, i.e. we have 1,605 weighting vectors.
  - For each combination of threshold k and weighting vector $\mathbf{w}$ ranks for all countries were computed.

Simulation of a wide range of *k* and **w** combinations



Empirical strategy, two analyses:

1. Statistical testing of dominance conditions.
2. Finding maximum change in weights that preserves the initial ranks.

For any sub-dimensional ratios we test:

$$Ho : z(r) = 0 \ \forall r = 1, 2, ..., R$$

$$Ha : z(r) < 0 \ \forall r = 1, 2, ..., R$$

Rejection of null: $\max\{z(1), z(2), ..., z(R)\} < z_\alpha < 0$.

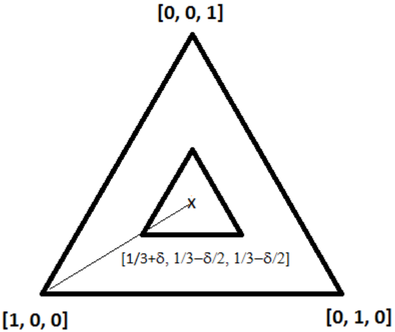
Test statistic:

$$T_{w,k} = \frac{\Pi^A(W,k) - \Pi^B(W,k)}{\sqrt{\frac{\sigma^2_{\Pi^A(W,k)}}{N^A} + \frac{\sigma^2_{\Pi^B(W,k)}}{N^B}}},$$

where:

$$\sigma^2_{\Pi^A(W,k)} = \Pi^A(W,k)[1 - \Pi^A(W,k)]$$

- **Q:** *How far can we go from equal weights while preserving the initial ranks in pair-wise comparisons?* (Permanyer, 2011)
- **The metric:** $\delta_{\max} = \max\{\delta\}$ s.t. $\nexists$ reranking , $\delta_{\max} \in \left[0, \frac{2}{3}\right)$



The following algorithm was used:

1. For each of the countries an initial ranking is detected for equal weights for each value of $k, k \in \{\frac{1}{240}, \frac{2}{240}, \ldots, 1\}_{N=240}$.
2. For each value of k the weights are redistributed: increasing the weight of one of the dimensions by δ and decreasing weights of the remaining dimensions by $\frac{\delta}{2}, \delta \in \{\frac{1}{120}, \frac{2}{120}, \ldots, \frac{2}{3}\}$, i.e. e.g.: $w_{1_i} = \frac{1}{3} + \delta_i, w_{2_i} = w_{3_i} = \frac{1}{3} - \frac{\delta_i}{2}$ for $i = 1, 2, \ldots, 80$.
3. For each pair of countries and each value of k a maximum value of δ which preserves the initial rankings (for that particular k) is identified.

Statistical evidence for dominance:

Table: Proportions of dominant pair-wise comparisons [%]

	04	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**	**13**
All data sets	52	51	44	47	42	42	49	56	56	59
Complete sets	x	x	x	x	42	42	47	54	55	55

Results for 2004 data:

AT		AT	AT	AT		AT	AT		AT			AT		
	DK		DK		DK	DK	DK	DK		DK				
	FI		FI		FI		FI	FI		FI				
			FR											
IS	IS	IS	IS	IS	IS	IS	IS	IS		IS		IS	IS	IS
LU	LU		LU	LU	LU		LU	LU		LU			LU	
	NO		NO		NO		NO	NO		NO				
	SE		SE				SE	SE		SE				
Countries above dominate the highlighted countries below														
AT	**BE**	**DK**	**EE**	**EL**	**ES**	**FI**	**FR**	**IE**	**IS**	**IT**	**LU**	**NO**	**PT**	**SE**
Countries below are dominated by the highlighted countres above														
									AT		AT			
BE		BE				BE			BE		BE	BE		BE
									DK					
EE		EE				EE	EE		EE		EE	EE		EE
EL									EL		EL			
ES		ES				ES			ES		ES	ES		
		FI							FI					
FR		FR				FR			FR		FR	FR		FR
IE		IE				IE			IE		IE	IE		IE
IT		IT				IT			IT		IT	IT		IT
									NO					
PT									PT		PT			
									SE					

Several dominance patterns can be identified. Based on the “longest path” we have e.g. the following clear dominance patterns:

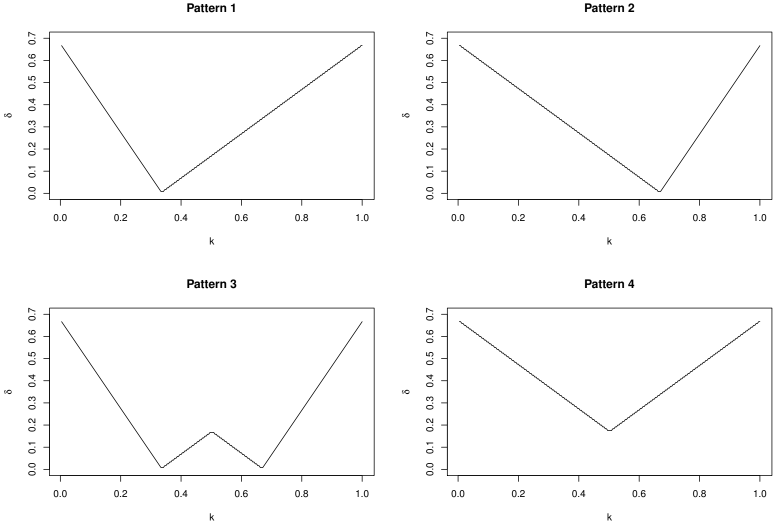
$$\text{(IS-NO-SE)} \rightarrow \text{(AT)} \rightarrow \text{(FR)} \rightarrow \text{(MT)} \rightarrow \text{(EE)} \rightarrow \text{(LT)} \rightarrow \text{(LV)} \rightarrow \text{(BG)}$$

$$\text{(IS-NO)} \rightarrow \text{(NL)} \rightarrow \text{(FI)} \rightarrow \text{(MT)} \rightarrow \text{(EE)} \rightarrow \text{(LT)} \rightarrow \text{(LV)} \rightarrow \text{(BG)}$$

For pairs of countries where dominance can not be assumed (in terms of the official EU multidimensional poverty indicator), four main patterns of relationship between $\delta_{\max}$ and k have been identified, and they account for over 90 % of all identified patterns.

What are the patterns?

RESULTS: Cross-country Comparisons (EU)
2. δ's vs k's: patterns



Conclusions

- **Work in progress!**
- We derive dominance conditions to test the robustness of comparisons for the Alkire and Foster's family of poverty measures.
- Easy to apply. Cross-country and cross-years analyses: about 50 % of the comparisons are not robust.
- Poverty orderings are very sensitive to weights and cut-off values.
- Important for the analysis of time trends and cross-country comparisons: more attention should be given to sensitivity analyses.

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